Picking Apples

This problem gives you the chance to:
• work out costs from given rules

Anna goes to pick apples.
She sees two orchards next to each other; David's orchard and Pam's orchard.
The signs below are at the entrance to the orchards.

DAVID'S APPLE ORCHARD Pick your own apples!	PAM'S ORCHARD DELICIOUS APPLES
First 10 pounds $2 per pound	$10 entry fee
Each additional pound $1 per pound	First 10 pounds $1.50 per pound
	Each additional pound $0.75

Anna wants to pick 40 pounds of apples.

1. a. How much does this cost at David's orchard? _____

   Show your calculations.

   b. How much does it cost at Pam's orchard? _____

   Show your calculations.

Chris has $30 to spend.

2. a. How many pounds of apples will he get if he goes to David's orchard? _____
Explain how you figured it out.

_____

_____

_____

b. If Chris goes to Pam's orchard, how many pounds of apples will he get? _____
Explain how you figured it out.

_____

_____

_____

3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?
Show your work. _____

10

Picking Apples	Rubric	
The core elements of performance required by this task are: • work out costs from given rules Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. a. Gives correct answer: $**50** Shows correct work such as: 10 x $2 + 30 x $1 b. Gives correct answer: $**47.50** Shows correct work such as: $10 + 10 x $1.50 + 30 x $0.75	1 1 1 1	4
2. a. Gives correct answer: **20** pounds Gives a correct explanation such as: The first 10 pounds of apples cost $20. The remaining $10 buys 10 pounds. Altogether 10 + 10 = 20 pounds. b. Gives correct answer: $\mathbf{16^2/_3}$ pounds (accept 16) Gives a correct explanation such as: The entry fee is $10. The first 10 pounds of apples cost $15. The remaining $5 buys 6.6 (accept 6) pounds. Altogether 10 + 6.6 = 16.6 pounds (accept 16)	1 1 1 1	4
3. Gives correct answer: **more than 30** pounds (Accept 31) Shows work such as: David's: 10 x $2 + 20 x $1 = $40 Pam's: $10 + 10 x $1.50 + 20 x $0.75 = $40 **or** Draws a correct graph	1 1 or 1	2
Total Points		**10**

Looking at Student Work on Picking Apples

Student A shows a clear understanding of the proportional relations described in "per pound" by showing the multiplication for the different amounts of apples. The student uses labels clearly to define what each computation represents. To solve for part 3 the student makes an organized list to show where David's cost is less than Pam's, at what point the costs are the same, and the where Pam's becomes less expensive.

Student A

Anna goes to pick apples.
She sees two orchards next to each other; David's orchard and Pam's orchard.
The signs below are at the entrance to the orchards.

DAVID'S APPLE ORCHARD Pick your own apples!	PAM'S ORCHARD DELICIOUS APPLES
First 10 pounds $2 per pound	$10 entry fee First 10 pounds $1.50 per pound
Each additional pound $1 per pound	Each additional pound $0.75

Anna wants to pick 40 pounds of apples.

4

1. a. How much does this cost at David's orchard? $50 ✓ ✓ 1

Show your calculations.

10 × 2 = $20 for first 10 lbs.

40 − 10 = 30 lbs. left

30 lbs × $1 per lb. = $30

$30 + $20 = $50 ✓ 1

b. How much does it cost at Pam's orchard? $47.50 ✓ ✓ 1

Show your calculations.

$1.50 × 10 = $15 for first 10 lbs.

30 lbs. left × .75 per each additional lb. = $22.50 ✓

$15 + $22.50 = $37.50

$37.50 + $10.00 entrance fee

$47.50

Student A, part 2

2. a. How many pounds of apples will he get if he goes to David's orchard? 20 pounds ✓ 1
Explain how you figured it out.

The first 10 pounds is $20, because 10lbs × $2 = $20. Chris then only ✓
has $10 left to spend, and each additional pound after buying ✓
the first 10lbs. is $1. So he can buy 10lbs of apples.

b. If Chris goes to Pam's orchard, how many pounds of apples will he get? 16 pounds ✓✓ 1
Explain how you figured it out.

$10 of Chris' money is spent on the entrance fee. So, he only has ✓ 1 ✓
$20 to spend on apple at Pam's orchard. The first 10lbs. at
Pam's costs now $15. He has $5 left to spend, but apples are 75¢
now that he has bought his first 10lbs. If he buys 6 more pounds he will
spend $4.50. (6 × .75 = 4.50) And he will spend a total of $29.50

3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?
Show your work.

31 pounds ✓ 1 ✓ 1

Davids	
lbs	$
19	29
20	30
21	31
25	35
30	40
31	41

Pams	
lbs	$
19	31.75
20	32.75
21	38.19
25	36.25
30	40
31	40.75

19 31.75

1		10
10	10.00	15
+15	+15.00	7.50
15	15.75	
40		

Student B is able to find the break-even point, where the cost for each person is the same. Then the student shows that at any point after 30 lbs. Pam's cost would be less. Student B has that sense than apples can be bought in pounds or fractions of a pound.

Student B

3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?
Show your work.

more than 30lbs ✗

davids
10lbs = $20
15lbs = $25
30lbs = $40

Pam's
10lbs = 25$
15lbs = 28.75
30lbs = $40 ✓✓

2

Student C approaches the problem from a different perspective by identifying the cause for Pam's initial higher costs, i.e. the entrance fee. The student then shows how many pounds must be purchased to compensate for that initial cost.

Student C

3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

   Show your work.

31

Pam must make up the $10 difference (the entrance fee),
First 10 lbs gives $5, after that you save 25¢ per lb,
5 multiplied by 4 (25¢ is ¼ of dollar) is 20, 20+10 is 30,
You must buy 31 lbs. before Pam's is cheaper.

Student D tries to use the organized list to find where Pam's cost is less than David's. The student knows that the change occurs between 30 lb. and 35 lb. The student's logic breaks down by forgetting that he is looking for the "smallest" amount where Pam's is cheaper rather than any amount where Pam's is cheaper. When looking at the table of values for 30 lb. and 35 lb., the student should have jumped to 31lb.

Student D

3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

   Show your work.

34lbs

unds	25lbs	30lbs	35lbs	34lbs
vid's	$35	$40	$45	$44
Pams	$36.25	$40	$43.75	$43

$20+$15 =35

$20+20 = $40 (10lb)

$10+(1.5x10)+15x.75 = 10+15+11.25 = 36.25

10+15+15 = 40

20+25 = 45

$10+15+8.75

$10+15+18 =

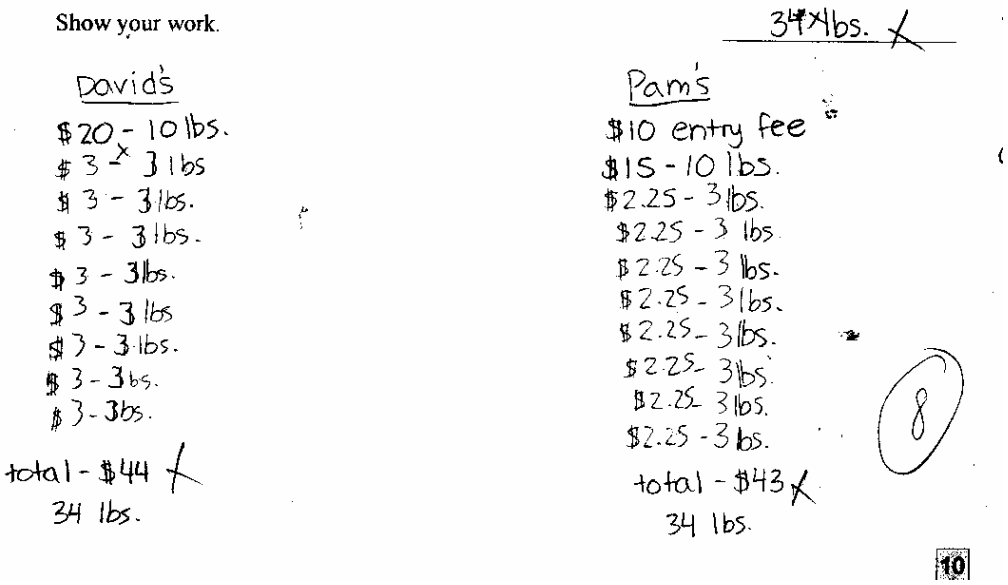
$20+24 = 44

Student E and Student F don't appear to try to narrow to the smallest amount of apples, where picking at Pam's is a better deal. They seem to be content with any value that will yield the desired results.

Student E

3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

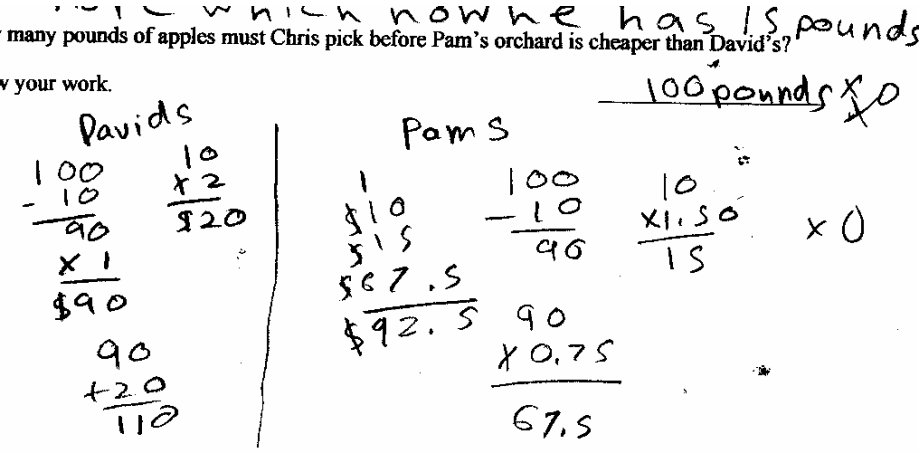
Show your work.



Student F

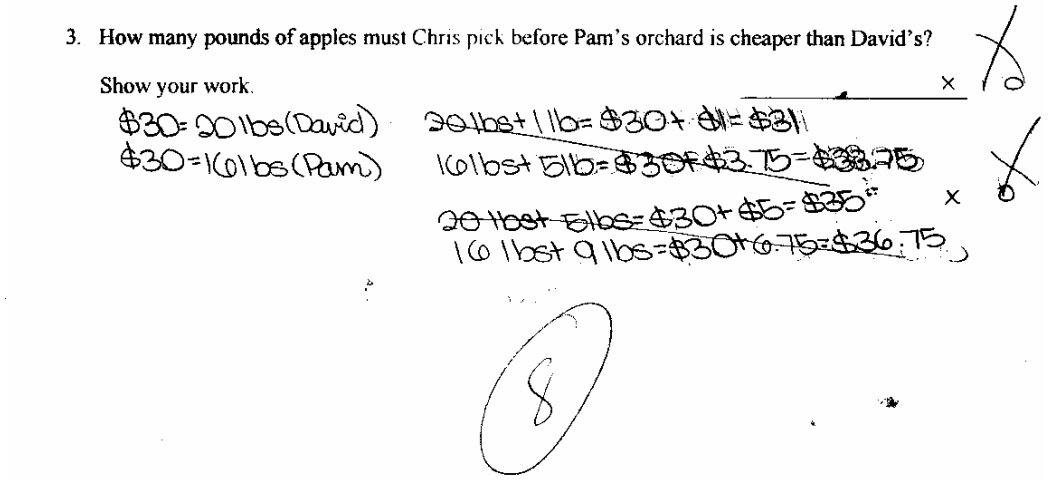
3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

Show your work.



Many students, including almost half the students with scores of 8/10, left part 3 blank. They did not have any strategies for how to approach the problem. Student G has managed to make the amount of money the same, but has different amounts of apples. The key to the question is to find the point where costs are the same for the same weight of apples.

Student G


3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

Show your work.

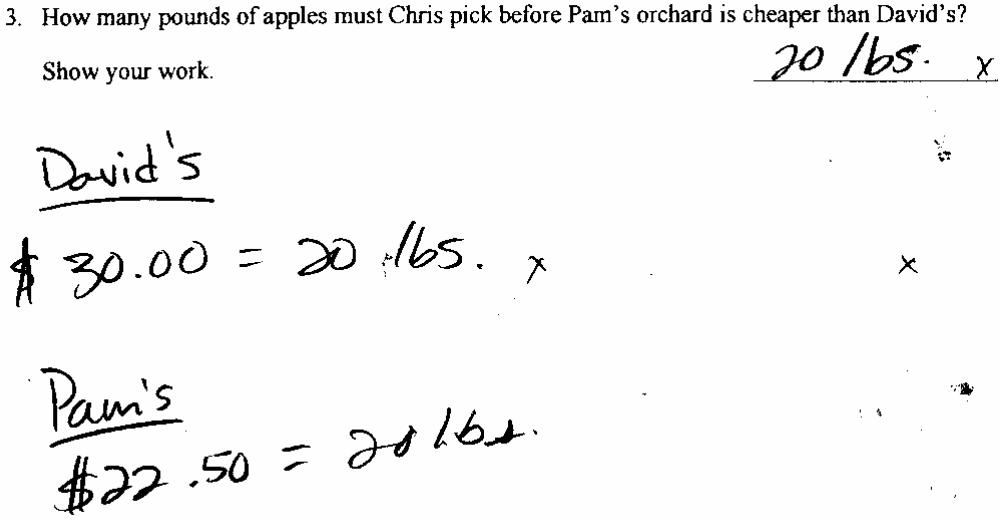
$30 = 20 lbs (David)
$30 = 16 lbs (Pam)

20 lbs + 1 lb = $30 + $1 = $31
16 lbs + 5 lb = $30 + $3.75 = $33.75
20 lbs + 5 lbs = $30 + $5 = $35
16 lbs + 9 lbs = $30 + 6.75 = $36.75

8

Student H forgets to use all the constraints: different costs for the first 10 lbs. and entry fee. Therefore the solution is incorrect.

Student H


3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

Show your work.

20 lbs.

David's

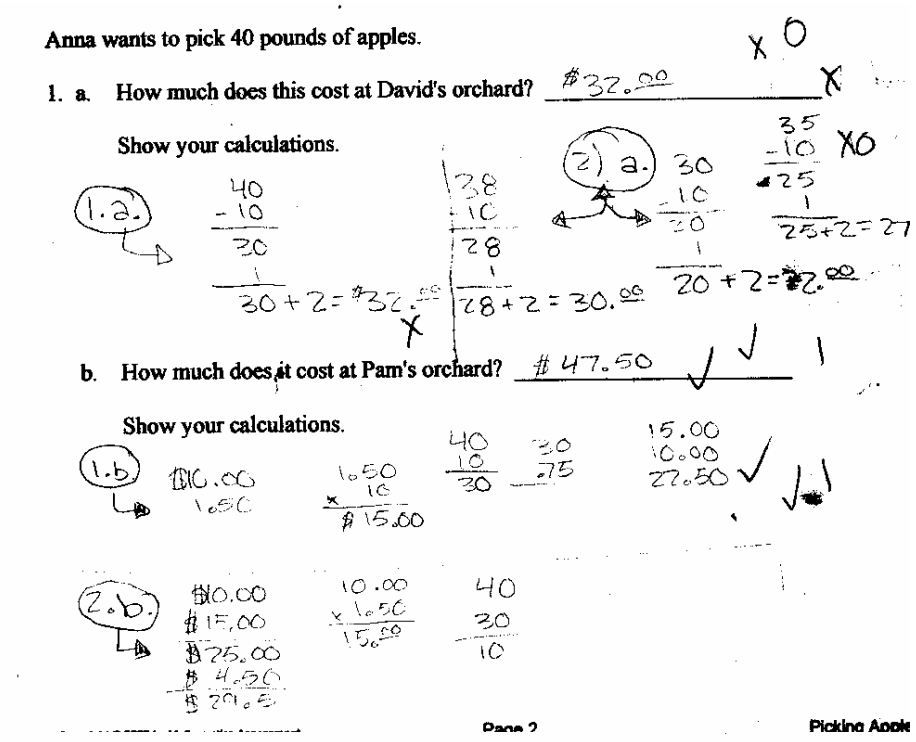
$30.00 = 20 lbs.

Pam's

$22.50 = 20 lbs.

Student I has difficult understanding the proportionality of $2 per pound in David's Orchard, making errors in both 1a and 2a in finding the costs. However the student is able to understand and use it appropriately to find the costs of Pam's orchard in 1b. In part 2b, the student is on the right track for finding the number of apples Pam can buy with $30, however the student forgets to label the + 6 as pounds of apples and adds 6 lbs. to $25 instead of 6 lb. and 10 lbs. Many students had similar difficulty of adding items with different labels.

Student I



Anna wants to pick 40 pounds of apples.

1. a. How much does this cost at David's orchard? #32.00

Show your calculations.

1.a. 40 - 10 = 30; 30 + 2 = #32.00

38 - 10 = 28; 28 + 2 = 30.00

2) a. 30 - 10 = 20; 20 + 2 = #22.00

35 - 10 = 25; 25 + 2 = 27

b. How much does it cost at Pam's orchard? #47.50

Show your calculations.

1.b. $10.00 + 1.50 = $15.00

40 - 10 = 30; 30 x .75

15.00 + 10.00 + 27.50

2.b. $10.00 + $15.00 + $25.00 + $4.50 = $29.50

10.00 x 1.50 = 15.00

40 - 30 = 10

Page 2 Picking Apple

Chris has $30 to spend.

2. a. How many pounds of apples will he get if he goes to David's orchard? 28 pounds

Explain how you figured it out.

The first 10 pounds are two dollars, so all you have to do is 38 - 10 = 28 + 1 = 28 + the two dollar for the first 10 pounds 28 + 2 = #30.00 shown on the first page 1.a.

b. If Chris goes to Pam's orchard, how many pounds of apples will he get? 31 pounds

Explain how you figured it out.

There is a 10 dollar entry fee, so that takes 10 dollar from the 30 dollars. The the First 10 pounds are 1.50 per pound. $10.00 + $15.00 = $25.00 + 6 more apples ($4.50) shown on the first page. 1.b.

Student J also has trouble with the dimensionality of the numbers. The student adds 10 lbs. and $20 to get 30 lbs. in 2a.

Student J

Chris has $30 to spend.

2. a. How many pounds of apples will he get if he goes to David's orchard? 30 lbs. X 0

Explain how you figured it out.

The first 10 lbs cost $20 and then he had $10 left and the additional lbs. cost $1 each. ✓ 1

b. If Chris goes to Pam's orchard, how many pounds of apples will he get? 10lbs ✓ 1

Explain how you figured it out.

He can buy $20 worth of apples. He spends $15 on the first 10 lbs and he can buy 6 more apples with the remaining $5. X 0

20 lbs to 10 $ 20

3. How many pounds of apples must Chris pick before Pam's orchard is cheaper than David's?

Student K has trouble understanding the constraints of the task. The student does not understand the proportional language of "per pound". In part 1 the student knows to multiply the additional pounds by $1, but treats the first 10 lb. as 1 unit costing $2, rather than $2 per pound. In part 1b, the student again understands the second quantity should be multiplied but ignores the entrance fee and cost of the first 10 pounds. In part 2 the student reasons backwards from his answer in part 1.

Student K

Anna wants to pick 40 pounds of apples.

1. a. **How much does this cost at David's orchard?** $32.00 X

Show your calculations.

40
-10 (Fib)
30 • 1 = 30 + 2 X

b. **How much does it cost at Pam's orchard?** $22.50 X

Show your calculations.

10 pds = 150
30
75
150
2100
2250

Chris has $30 to spend.

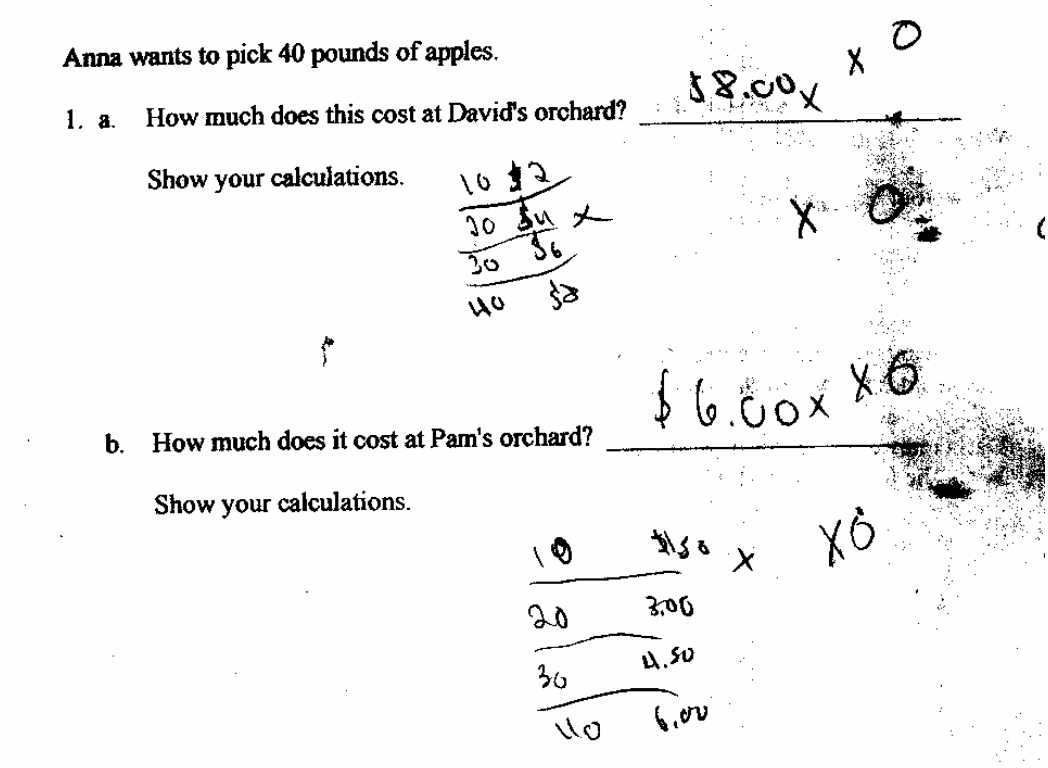
2. a. **How many pounds of apples will he get if he goes to David's orchard?** 38 apples X

Explain how you figured it out.

40 appls = $32.00 - $2.00 = 30 - 2 appls
= 38 appls. X

Student L has trouble with the idea of "per pound", thinking in terms of groups of tens instead of groups of 1. Notice also that the student does not change the cost after the first ten pounds.

Student L



Anna wants to pick 40 pounds of apples.

1. a. How much does this cost at David's orchard? $8.00 x 40

Show your calculations.

10 $2
20 $4 x
30 $6
40 $8

b. How much does it cost at Pam's orchard? $6.00 x 40

Show your calculations.

10 $1.50 x 40
20 3.00
30 4.50
40 6.00

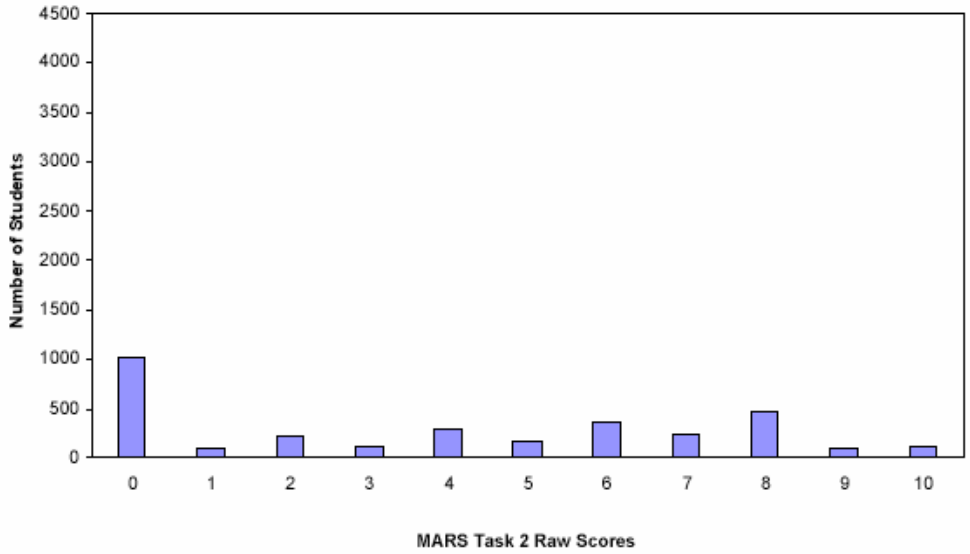
Teacher Notes:

MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 8

Task 2 - Picking Apples

Mean: 3.90 StdDev: 3.36

Figure 50: Bar Graph of MARS Test Task 2 Raw Scores, Grade 8



Score	0	1	2	3	4	5	6	7	8	9	10
Count	1015	97	227	122	384	167	352	233	455	96	122
%<=	32%	25.1 %	42.2%	46.1%	55.0%	60.3%	71.4%	78.8%	93.1 %	96.2%	100%
%>=	100%	68%	64.9%	57.8%	53.9%	45%	39.7%	28.6%	21.2 %	6.9%	3.8%

The maximum score available for this task is 10 points.
The minimum score for a level 3 response, meeting standards, is 5 points.

Many students, about 64% could find the cost of 40 lbs. of apples picked at David's orchard and show how they figured it out. About half the students could find the cost of 40 lb. of apples at David's orchard or work backwards from an amount of money to the number of pounds picked at David's. About 45% could also make some sense of picking 40 lb. of apples at Pam's orchard by either getting a correct solution or a correct process. Some students, about 21% could find the cost of 40 lb. of apples at either orchard and the number of pounds that could be purchased for $30 at either orchard. 3.8% of the students could meet all the demands of the task, including finding the lowest point where Pam's orchard is less expensive than David's including some justification to back up that answer. 32% of the students scored no points on this task. 92% of the students with this score attempted the task.

Picking Apples

Points	Understandings	Misunderstandings
0	92% of the students with this score attempted the task.	Some students had difficulty interpreting the language of proportionality, "per pound". 8% of all students had answers of $32 for apples at David's, because they didn't multiply the first 10 lbs. by $2. 8% of the students thought the cost of 40 lbs. was $40, just taking $1 for every pound and ignoring the difference in price for the first 10 lbs.
2	Students could interpret the meaning of "per pound" and calculate the cost of 40 lbs. of apples at David's orchard.	Students had difficulty interpreting the 3 constraints for Pam's orchard. 11% of the students ignored the entry fee. 4% only added $1.50 for the first 10 lbs. getting answer of $34. 5% did not use monetary notation giving an answer of $47.5.
4	Students could calculate the cost of buying 40 lbs. of apples at David's and find the number of pounds that could be purchased for $30, showing appropriate calculations for each.	11% of the students, who missed part 2a, ignored the change in cost for the first 10 lb. They thought $30 would get 30 lbs. 6% thought that if 40 lbs. = $40 and the first 10 lb. = $2, then $30 would buy 38 lbs. Other common answers for 2a were 15, 150, and 40 pounds.
6	Students could find the cost of 40 lbs. at both orchards. They could work backwards from a cost of $30 to the amount of apples purchased at David's, but not at Pam's.	Some students had difficulty working backwards from a cost of $30 to the number of pounds at Pam's. Some calculated it as if all apples cost $ .75, giving them an answer of 40 lb. Some students did not realize you can by fractional amounts of pounds, so they picked answers that would use most of the money like 13 or 15 pounds.
8	Students could find the cost of 40 lbs. or the number of pounds that could be purchased for $30 at both orchards, showing appropriate calculations.	Many students who were successful at all other parts of the task did not attempt the final part of the task. They did not know how to attack the problem. 65% of all students did not attempt this part of the task.
10	Students could find the cost of 40 lbs. or the number of pounds that could be purchased for $30 at both orchards, showing appropriate calculations. Students could also find the point where Pam's orchards were cheaper than David's.	Students might try guess and check to find some value where Pam's orchard was cheaper, but not narrow it down to the lowest amount giving answers as high as 100 or 187 lbs.

Based on teacher observations, this is what eighth graders knew and were able to do:

- They could multiply to find the costs of additional pounds

Areas of difficulty for eighth graders:

- Interpreting the language of proportions, "per pound"
- Identifying and using all the constraints in a problem
- Labeling answers to understand what had been calculated
- Adding together inappropriate items like pounds and dollars
- The concept of a break-even point, setting up an equality to find out when two alternatives will yield the same value
- Standard monetary notation

Questions for Reflection on Picking Apples

- What kinds of language do your students use to make sense of rates or proportional situations? Do you think they understand terms like per pound, per hour, per box? Do you think they see these terms as sets of equal size groups?
- What types of activities do students do to help them make sense of the meaning behind rates or proportions? Do they associate multiplication/ division with these ideas? In what ways?
- How are labels used in the classroom when solving problems? Do you provide explicit instruction to help students deal with dimensional analysis or how operations effect or change labels?
- Looking at student work in part 1 and 2, were students thinking in terms of function or doing several individual calculations?

Look at student work for part 1a. How many of your students put:

$50	$40	$32	$8	$30	$20	Other

Can you follow the reasoning chain that led to each particular error pattern? What does this show you about student misunderstandings?

Look at student work for 1b. How many of your students put:

$47.50	$47.5 or $47.05	$55	$34	$37.50	Other

What additional misconceptions contributed to the problems in this part of the task?

- What types of problems do students work requiring them to "work backwards" or do inverse operations? Do students work with this idea with computational procedures or just with problem-solving tasks?
- Are students comfortable with order of operations and how that works when undoing a procedure?

- How comfortable are students with symbolic notation like parentheses, division symbols?
- When looking at student work in part 2, check student thinking to see if they combined inappropriate terms like $ and pounds. Did they lose track of the meaning behind their computations?

Look at student work on 2a. How many students put:

20	15	30	150	38	40	Other

Look at student work on 2b. How many students put:

16 2/3 or 16	10	30	19	20	25	No response	Other

What misconceptions led to these error patterns? What made this part more difficult for students?

Now look at work in the final section. Many students did not know how to approach this part of the task. They did not have the sense of finding the point where the number of pounds and the cost were the same for both orchards.

- How many of your students did not attempt this part of the task?
- How many of your students guessed a large value (34 to 187) that made Pam's cheaper?
- How many lost track of some of the constraints (like entrance fee) when making their calculations for this section?
- Were successful students able to set up an equation to solve the problem or did they use guess and check to solve the problem?

Do you have any problems in your text dealing with the idea of break-even point? Have you students worked with problems graphing two equations to find where they intersect? What strategies would you have expected or wanted your students to be able to use?

Implications for Instruction:

Students at this grade level should be comfortable with identifying and using constraints to solve problems. Students should be starting to do operations with labels to keep track of how the calculations change the labels. Students should also start to use equations to express the multiple constraints, rather than using a string of calculations.

A big idea for middle grades is the ability to use proportional reasoning or understand multiplicative relationships. Students should be comfortable with the language of proportions or rates, like per pound, per hour, per person. Students need explicit instruction of help them connect cost per pound or miles per hour as representing equal groups that can be multiplied or divided. Students also need help seeing how these operations change the units.

Students at this grade level should be preparing for the transition to algebra. They need many opportunities to work problems involving inverse operations. They need to be confronted with situations with multiple steps, where order of operations makes a difference. So when students share solutions, it is important for them to be asked why they have different answers. The teacher might pose questions, such as, "If two students both subtracted and both divided, how is it possible for them to get two different solutions? How can we determine which one is correct?" Having students grapple with these issues helps them see the logic behind the rules or algorithms in a way that direct instruction alone doesn't.

Another big idea to help then prepare for algebraic thinking is the idea of equality. In order for students to think about the idea of when Pam's orchard is cheaper, it is helpful to ask the question, " At what point are the two orchard's the same?" As more than half the students had no idea how to even start this part of the task, giving them this as a discussion point when returning the papers would be a good classroom activity.

Teacher Notes:

Performance Assessment Task

Picking Apples
Grade 8

The task challenges a student to demonstrate understanding of the concepts representing and analyzing mathematical situations and structures using algebra. A student must understand and be able to represent a break-even point in cost analysis. A student must be able to use symbolic algebra, a table, a graph, or a verbal explanation to represent and explain mathematical relationships. A student must be able to judge the meaning, utility, and reasonableness of results of one or more of these representations. A student must make sense of this mathematics in a real world context of picking apples in two different orchards.

Common Core State Standards Math - Content Standards

Expressions and Equations

Analyze and solve linear equations and pairs of simultaneous linear equations..

8.EE.8 Analyze and solve pairs of simultaneous linear equations.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Common Core State Standards Math – Standards of Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Assessment Results

This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
8	2005	10	5	45%